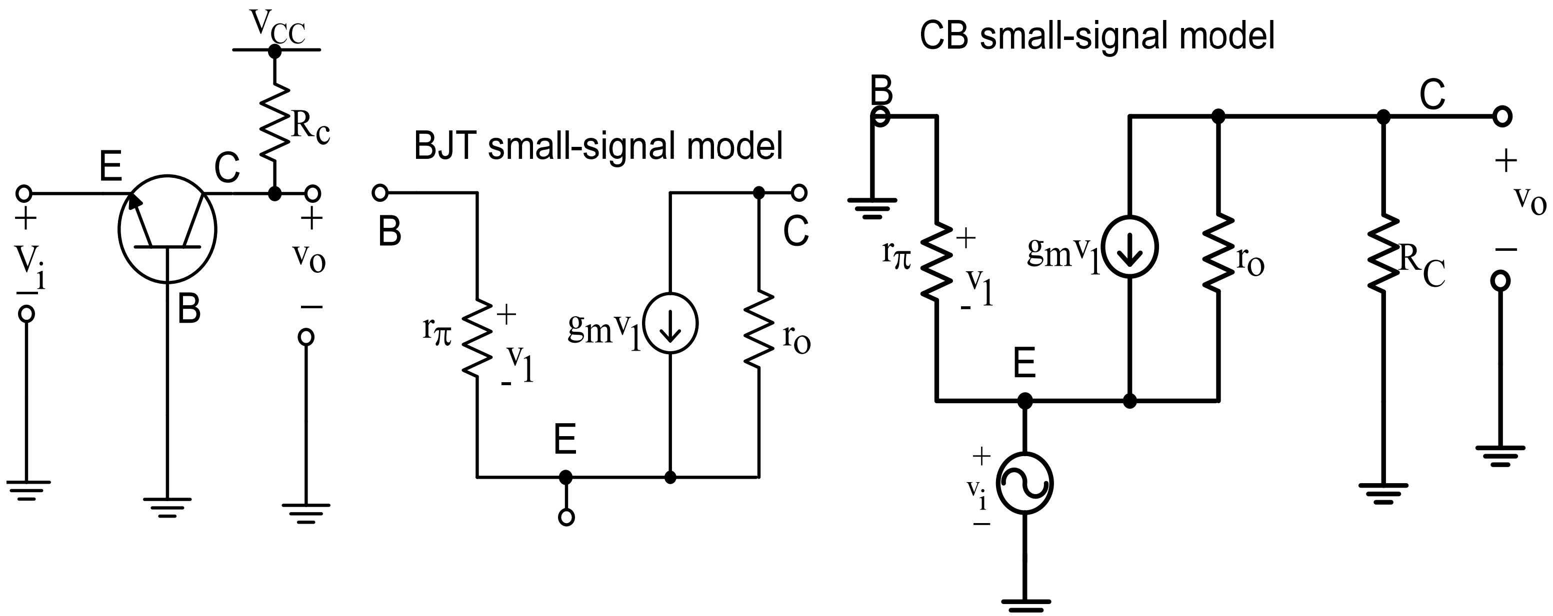
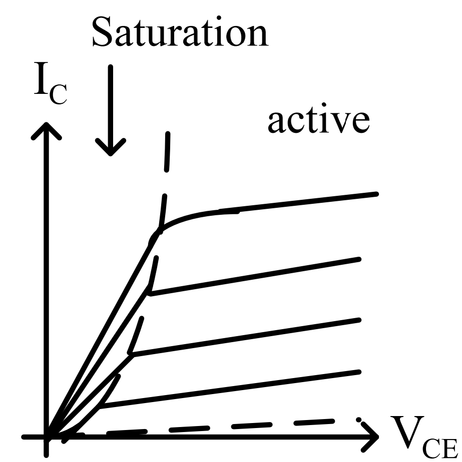
UNIVERSITI SAINS MALAYSIA

EEE 241
ANALOG ELECTRONICS
Class 5&6&7&8&9

DR NORLAILI MOHD NOH

3.3.3 Common-Base configuration



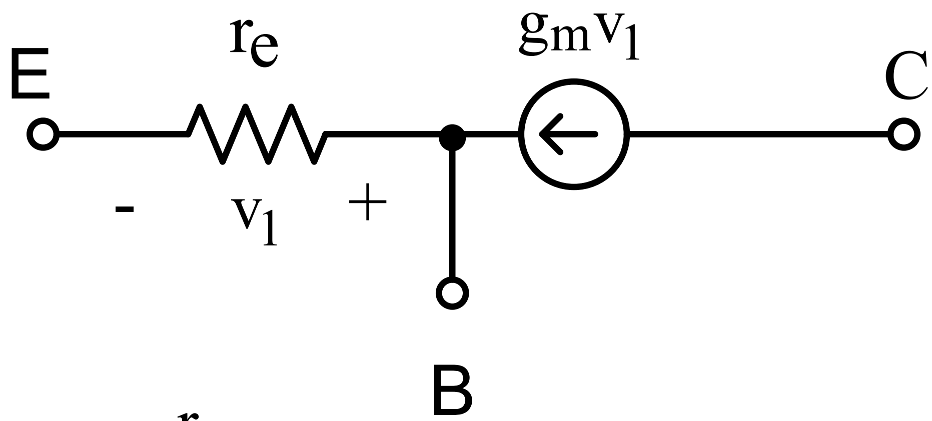


i/p signal applied to E. O/p taken from C. B tied to ac gnd.

The hybrid-π model provides an accurate representation of the small-signal behavior of the transistor independent of the circuit configuration. However, for the common-B, the hybrid-π model becomes tougher to analyze as the dependent current source is connected between the i/p and o/p terminals.

To simplify the analysis of a common-B (CB) amplifier, use a T-model instead.

The T-model at low freq:

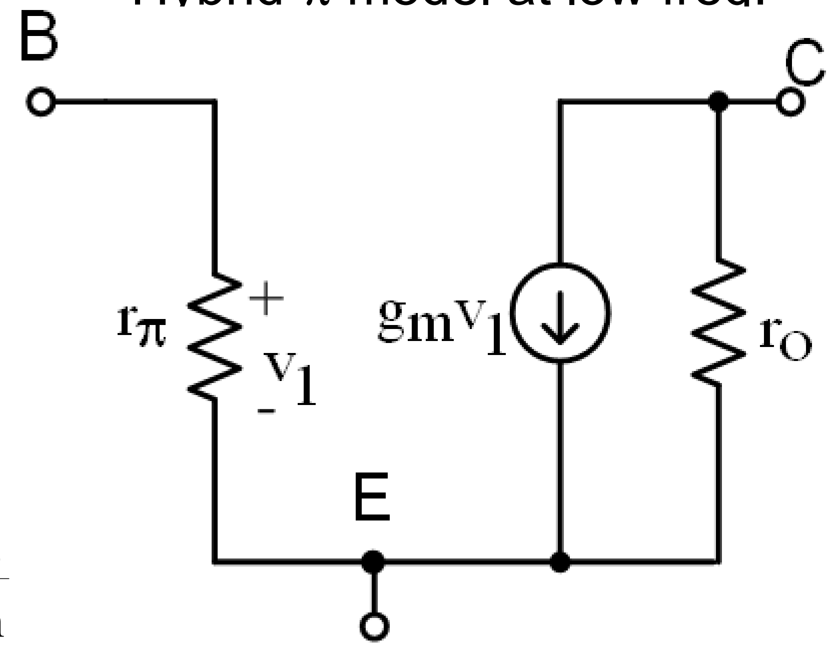


$$r_e = \frac{r_\pi}{1+g_m r_\pi}$$

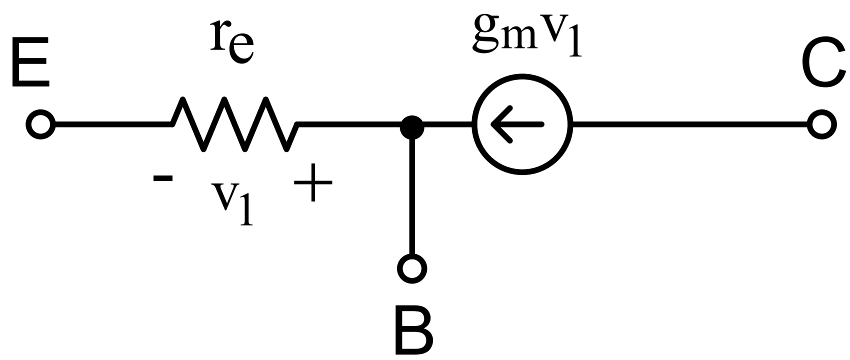
$$r_\pi = \frac{\beta_o}{g_m}$$

$$r_e = \frac{r_\pi}{1+g_m r_\pi} = \frac{\beta_o}{g_m\left(1+g_m\frac{\beta_o}{g_m}\right)} = \frac{1}{g_m}\frac{\beta_o}{1+\beta_o} = \frac{\alpha_o}{g_m}$$

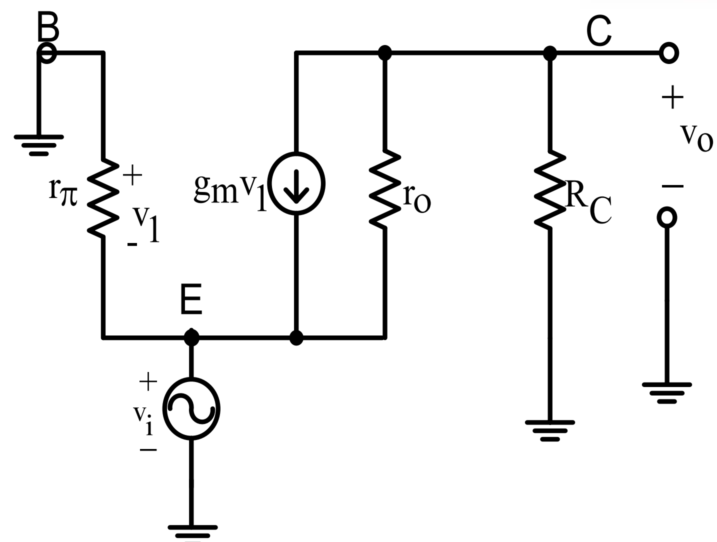
Hybrid-π model at low freq:



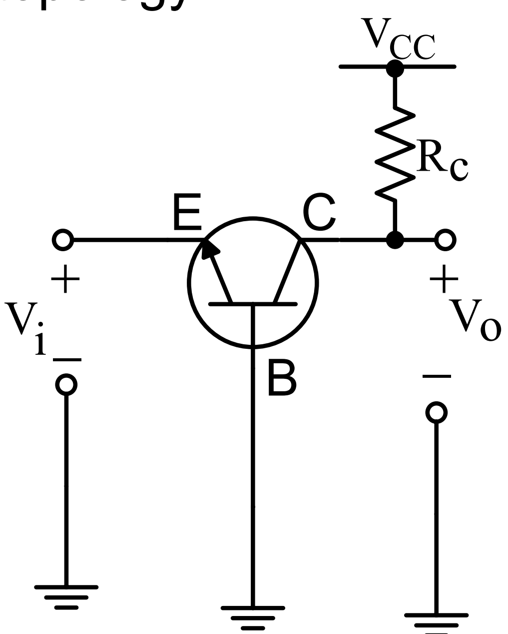
T-model:

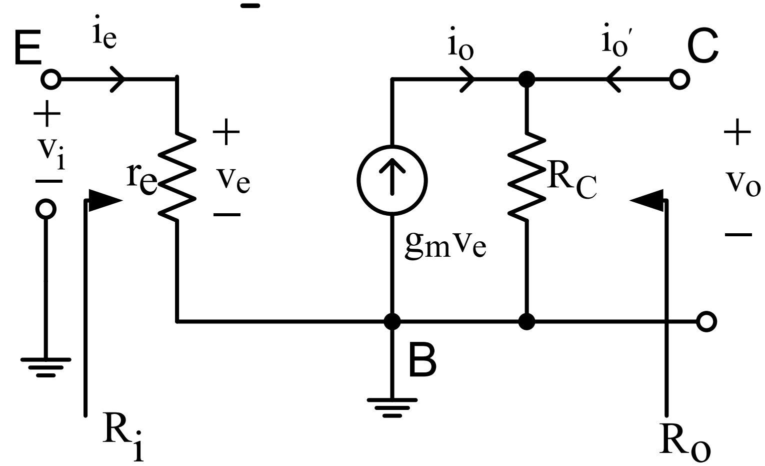


CB hybrid-π model

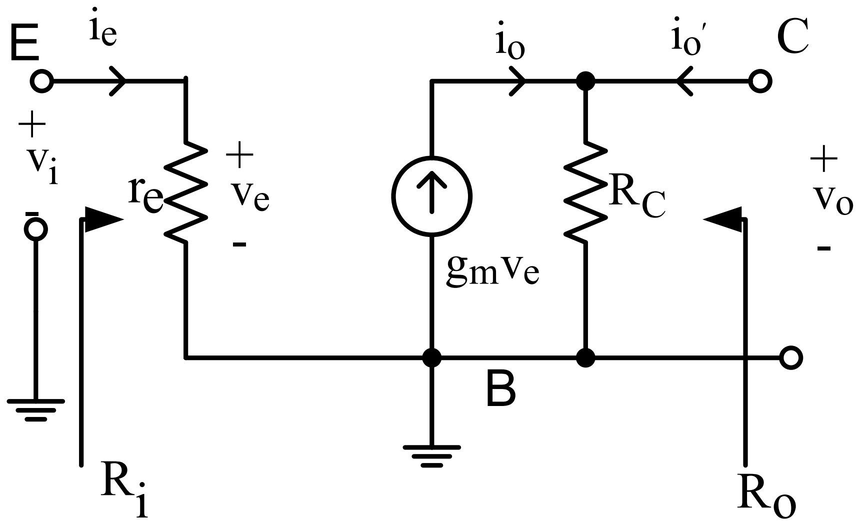


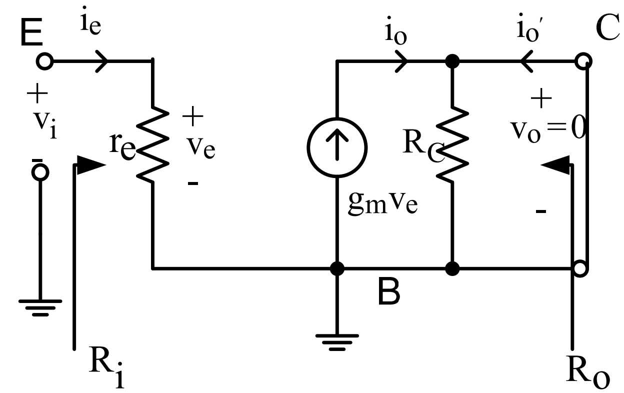
CB topology:





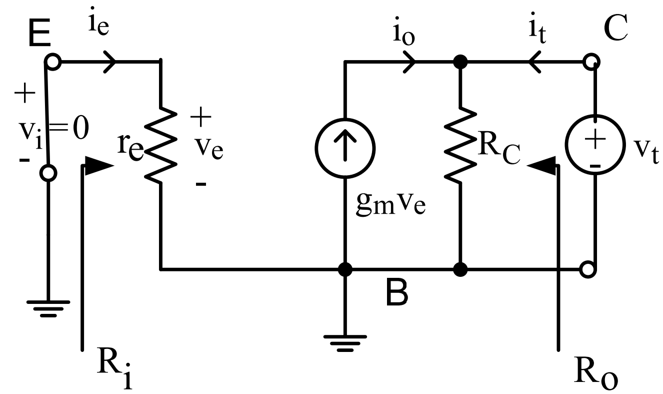
T-model of the CB topology:





The s/c transconductance: $G_m = \left.\frac{i_o}{v_i}\right|_{v_o=0} = \frac{g_m v_e}{v_e} = g_m$

The input resistance: $R_i = \frac{v_i}{i_e} = \frac{v_e}{i_e} = r_e$



The output resistance: $R_o = \left.\frac{v_t}{i_t}\right|_{v_i=0} = R_C$

o/c or unloaded voltage gain,

$a_v = \left.\frac{v_o}{v_i}\right|_{i_o'=0} = \frac{g_m v_e R_C}{v_e} = g_m R_C$ and $a_i = \left.\frac{i_o}{i_i}\right|_{v_o=0} = \frac{g_m v_e}{v_e / r_e} = g_m r_e$

$$a_i = \left.\frac{i_o}{i_i}\right|_{v_o=0} = \frac{g_m v_e}{v_e / r_e} = g_m r_e$$

Since, $r_e = \frac{\alpha_o}{g_m}$, then $a_i = g_m \frac{\alpha_o}{g_m} \approx 1$

For the CE configuration: $R_i = r_\pi = \frac{\beta_o}{g_m}$

For the CB configuration: $R_i = r_e = \frac{\alpha_o}{g_m}$

$R_{i\_CE} > R_{i\_CB}$

$$R_{i\_CB} = \frac{\alpha_o}{\beta_o} R_{i\_CE} = \frac{1}{1+\beta_o} R_{i\_CE}$$

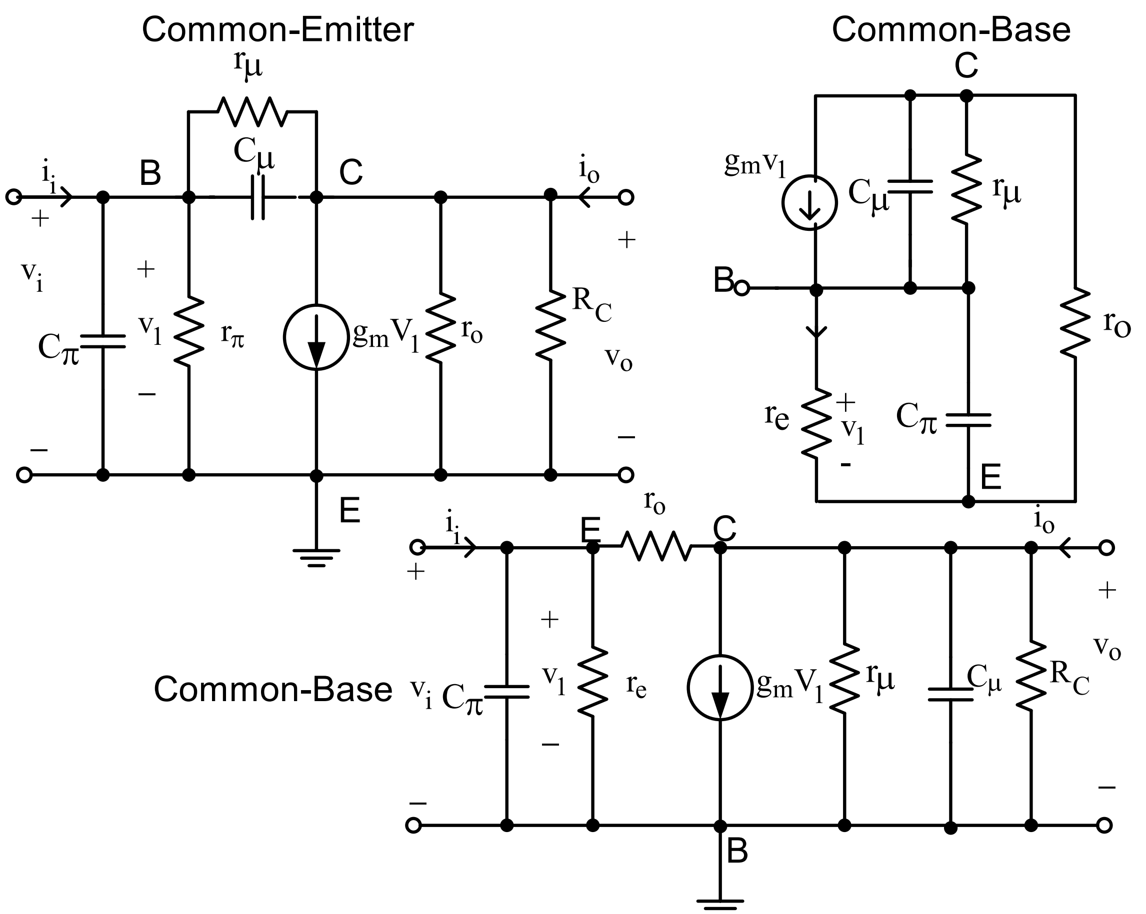
$$a_{i\_CB} \approx \alpha_o$$

$$a_{i\_CE} \approx \beta_o$$

$$a_{i\_CB} < a_{i\_CE}$$

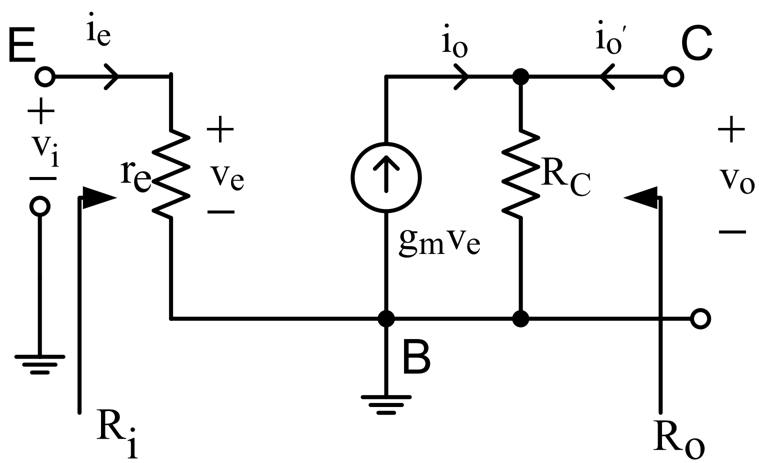
$$a_{i\_CB} = \frac{\alpha_o}{\beta_o} a_{i\_CE} = \frac{1}{1+\beta_o} a_{i\_CE}$$

In terms of i/p resistance and current gain, the CE amplifier performs better than CB.

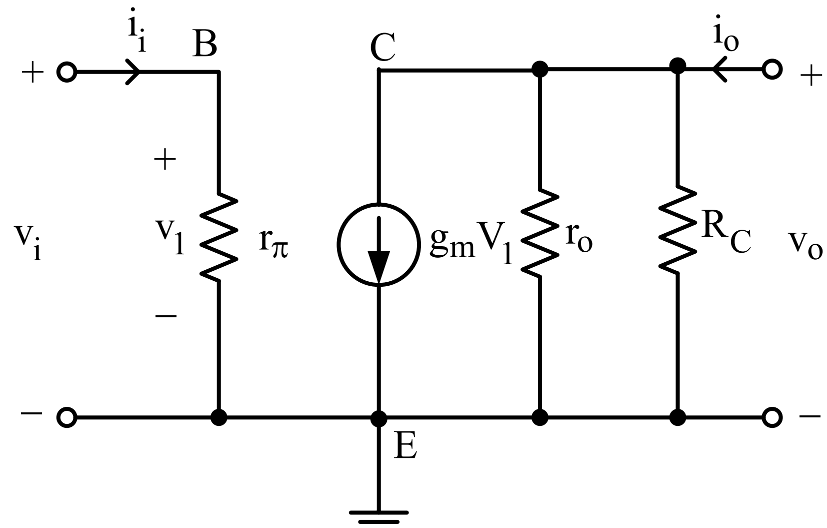


C_μ is between B-C. At high freqs., capacitive components are dominant.
For CE, C_μ is between i/p and o/p. Hence, at high-freqs., there will be a feedback from o/p to i/p.
For CB, i/p is at E and o/p is at C. Therefore, C_μ will not cause a feedback at high-freqs. CB circuits are used for high-freq. application.

CB configuration



CE configuration



In CB configuration, $R_o = R_C$

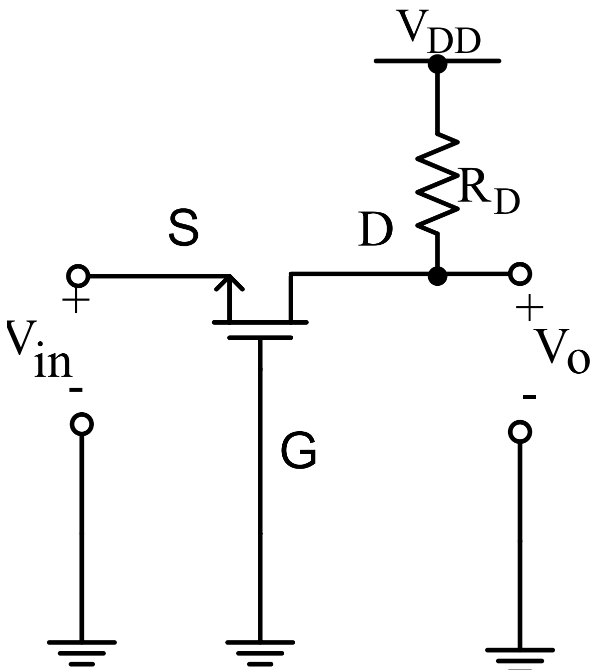
In CE configuration, $R_o = R_C \,//\, r_o$

If $R_C \to \infty$, $R_{o\_CB} \to \infty$ and $R_{o\_CE} = r_o$

Under this condition, $R_{o\_CB} > R_{o\_CE}$

Besides using the CB as high freq. amplifier, it can also be used as a current source whose current is nearly independent of the voltage across it (i.e. $i_o = g_m v_e$)

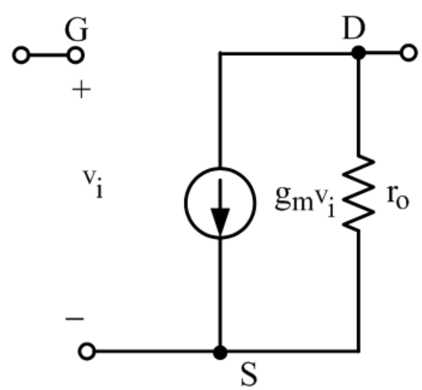
3.3.4 Common-gate (CG) configuration.



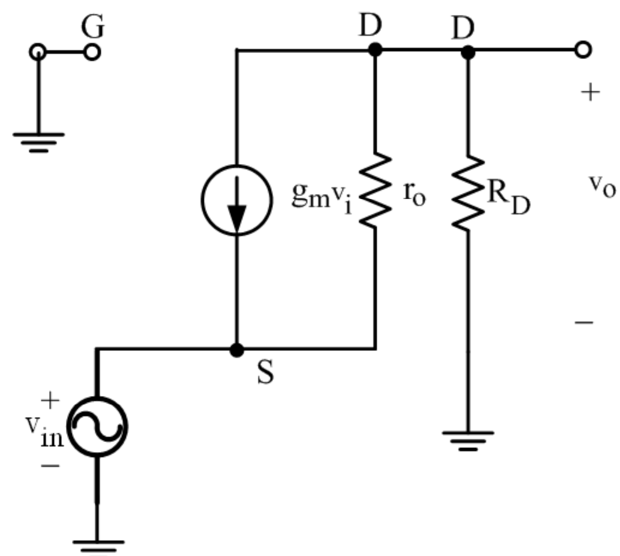
I/p signal is applied to the S. O/p is taken from the D. G is connected to the ac gnd.

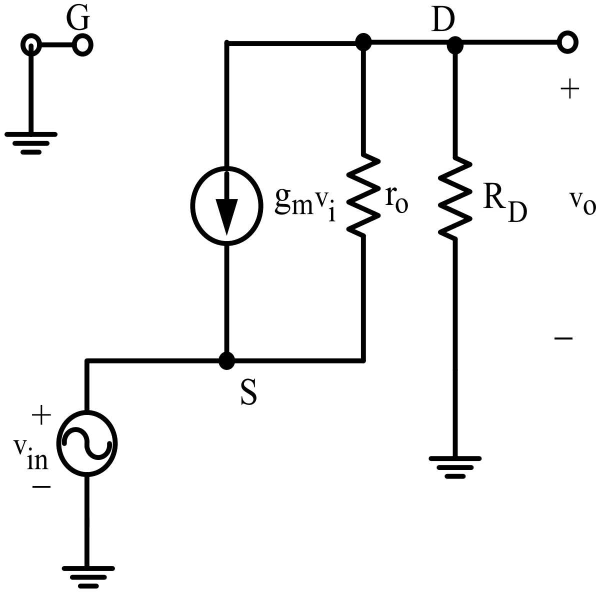
The analysis of CG amplifiers can be simplified if the model is changed from a hybrid-π to a T-model.

ac model for FET



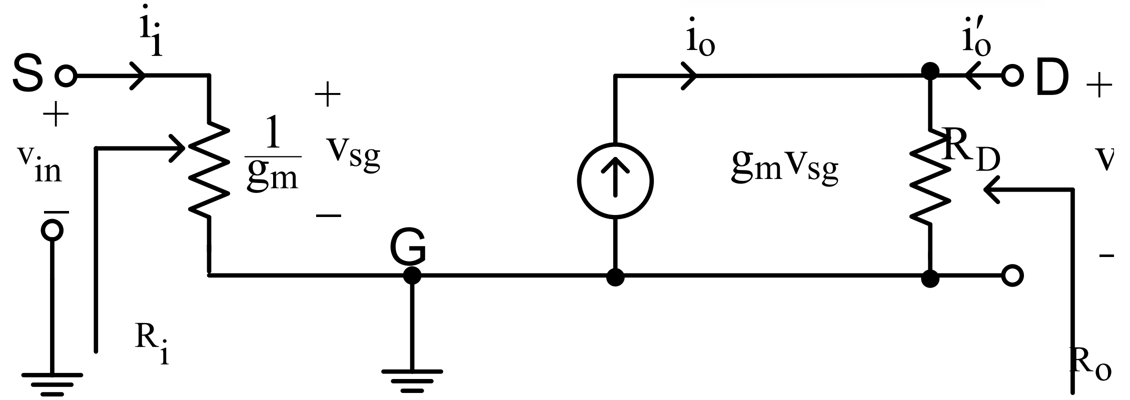
ac model for CG

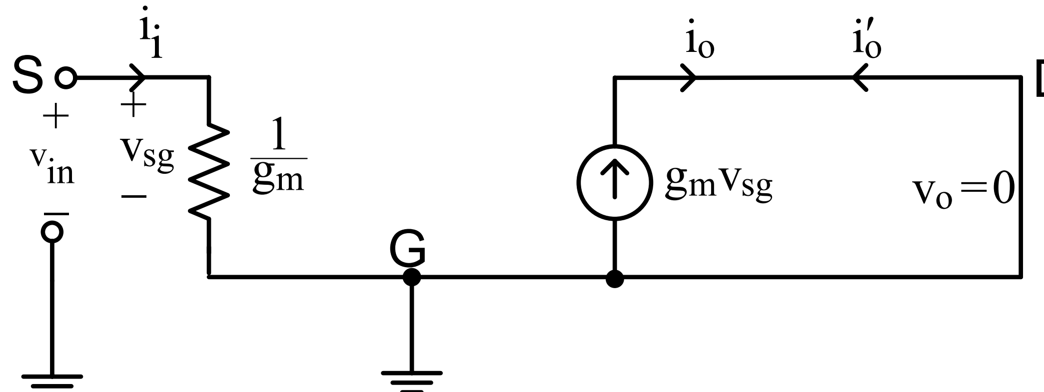


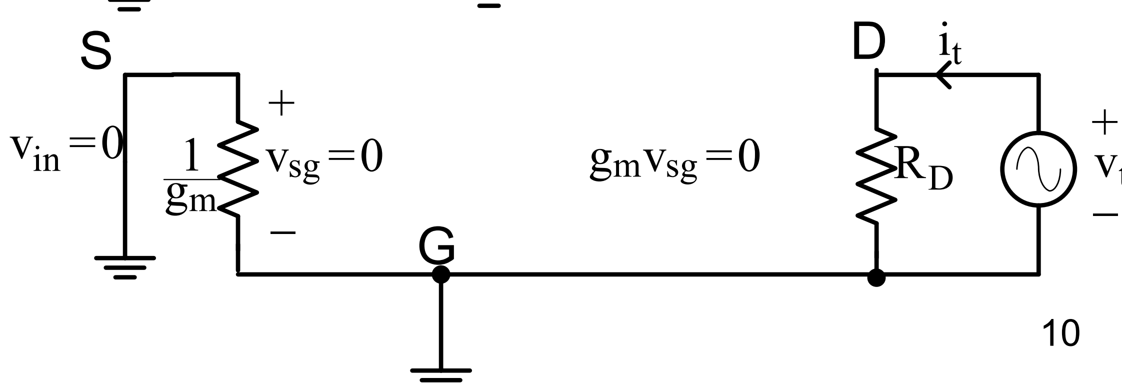


Hybrid-π model for FET

T-model for CG



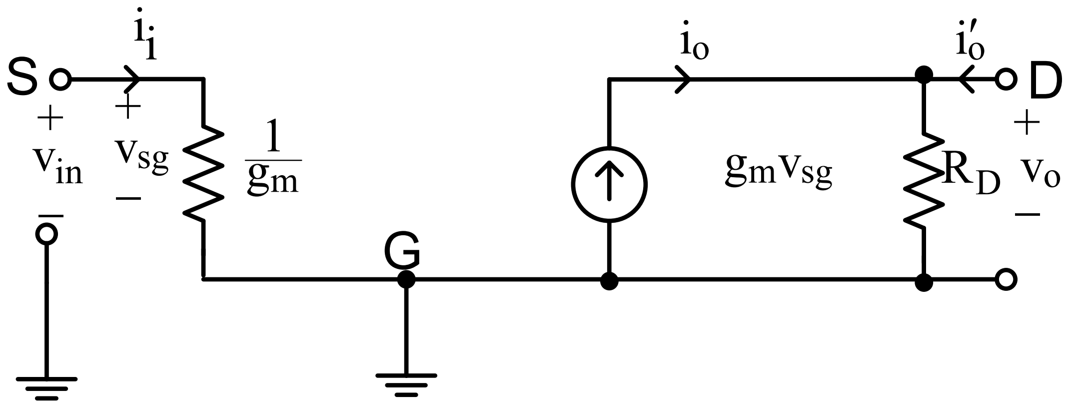




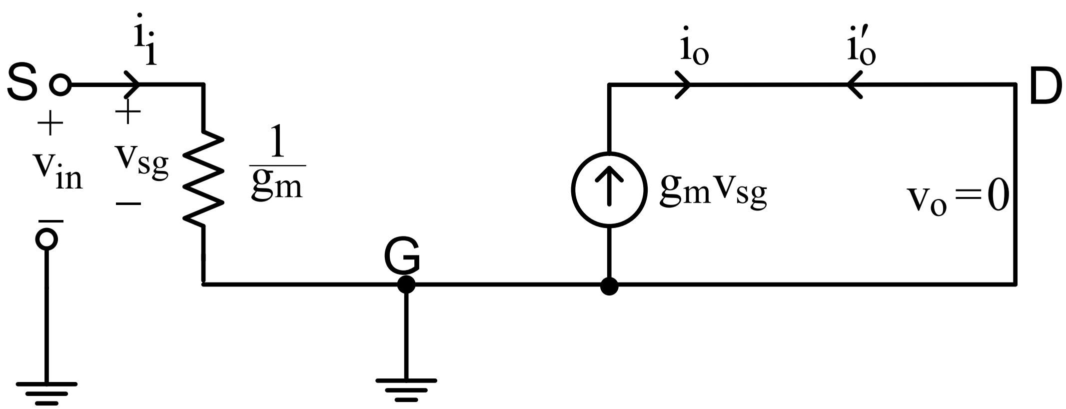
$$G_m = \left.\frac{i_o}{v_{in}}\right|_{v_o=0} = \frac{g_m v_{sg}}{v_{sg}} = g_m$$

$$R_i = \frac{v_{in}}{i_i} = \frac{v_{sg}}{i_i} = \frac{1}{g_m}$$

$$R_o = \left.\frac{v_t}{i_t}\right|_{v_i=0} = R_D$$

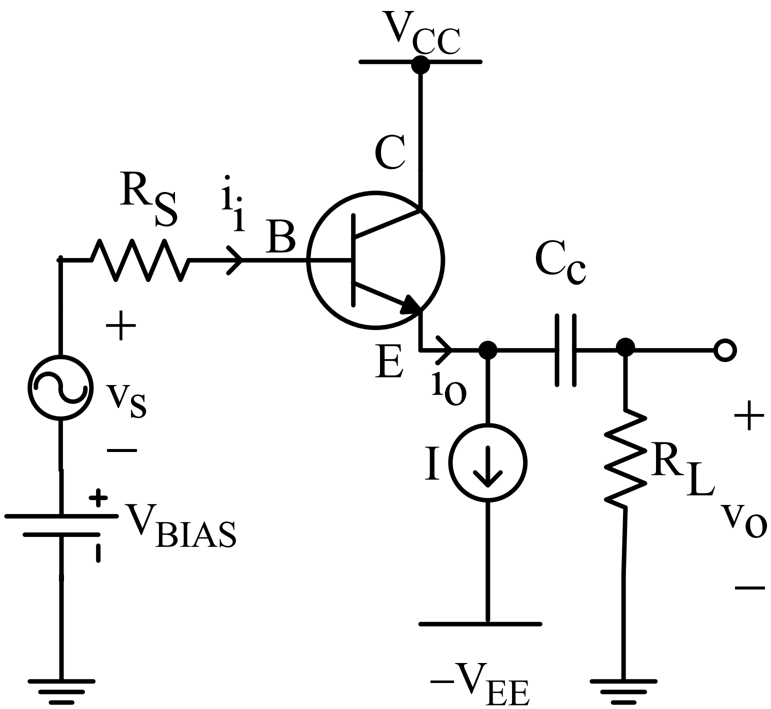


$$a_v = \left.\frac{v_o}{v_{in}}\right|_{i_o'=0} = \frac{g_m v_{sg} R_D}{v_{sg}} = g_m R_D$$

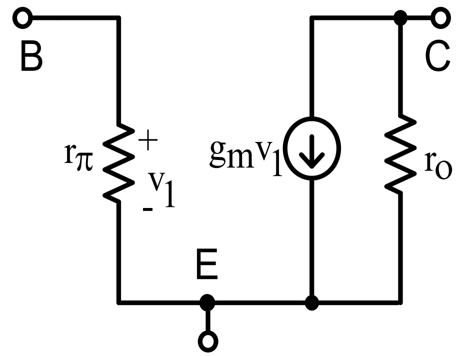


$$a_i = \left.\frac{i_o}{i_i}\right|_{v_o=0} = \frac{g_m v_{sg}}{\frac{v_{sg}}{\frac{1}{g_m}}} = g_m\left(\frac{1}{g_m}\right) = 1$$

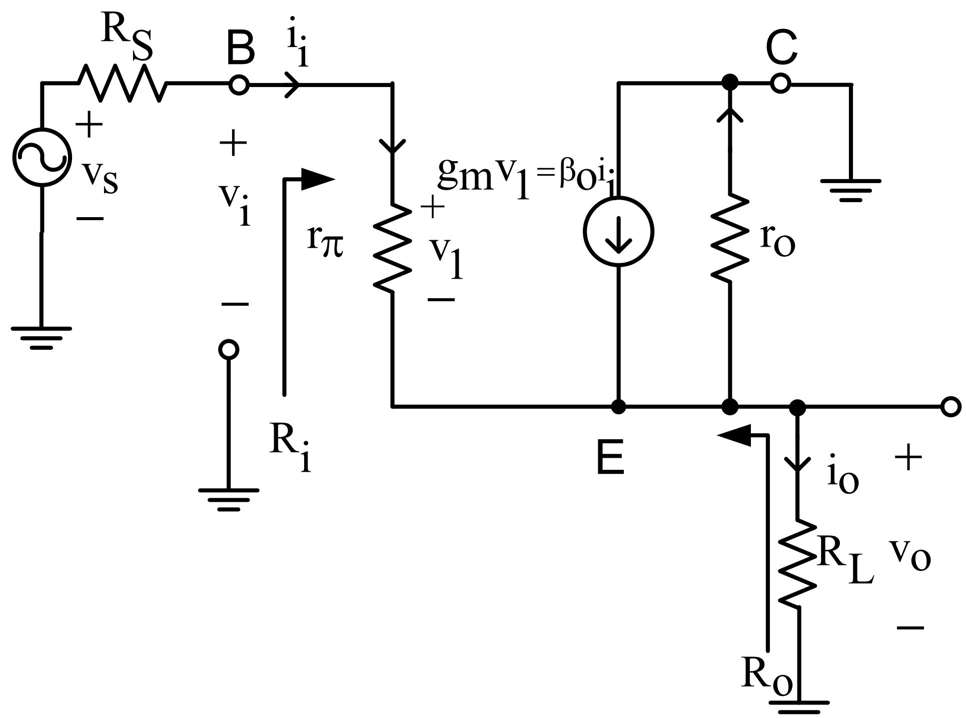
3.3.6 Common-collector (CC) configuration (Emitter follower)



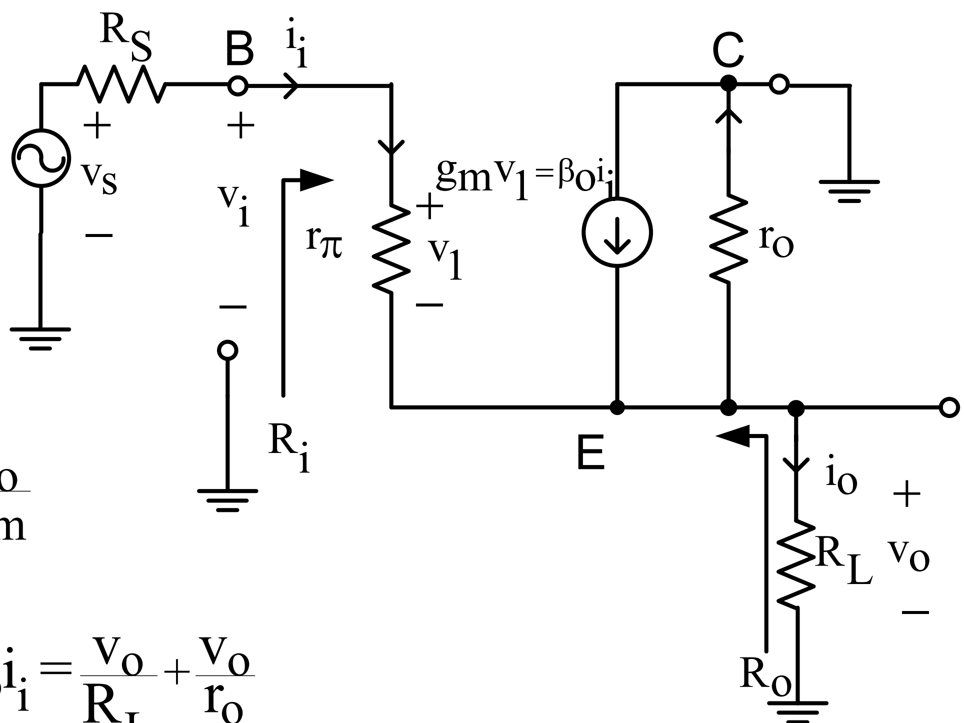
BJT small-signal model



Small signal/ac/hybrid-π model for a CC circuit



i/p signal applied to the B.
o/p signal taken from the E.



$$r_\pi = \frac{\beta_o}{g_m}$$

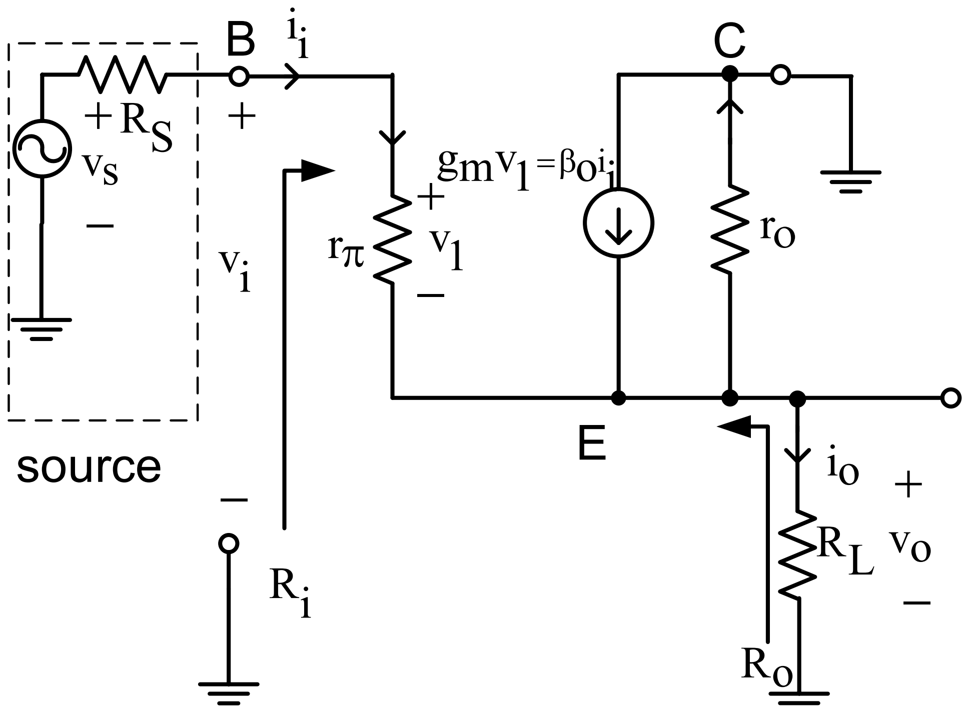
$$g_m v_1 = g_m i_i r_\pi = g_m i_i \frac{\beta_o}{g_m}$$
$$= \beta_o i_i$$

KCL at node E, $i_i + \beta_o i_i = \frac{v_o}{R_L} + \frac{v_o}{r_o}$

$v_S = i_i\,(R_S + r_\pi) + v_o$

$v_i = i_i r_\pi + v_o$

$R_i = v_i / i_i$



<u>To determine R_i :</u>

$R_i = v_i / i_i$

KCL at node E, $i_i + \beta_o i_i = \frac{v_o}{R_L} + \frac{v_o}{r_o}$

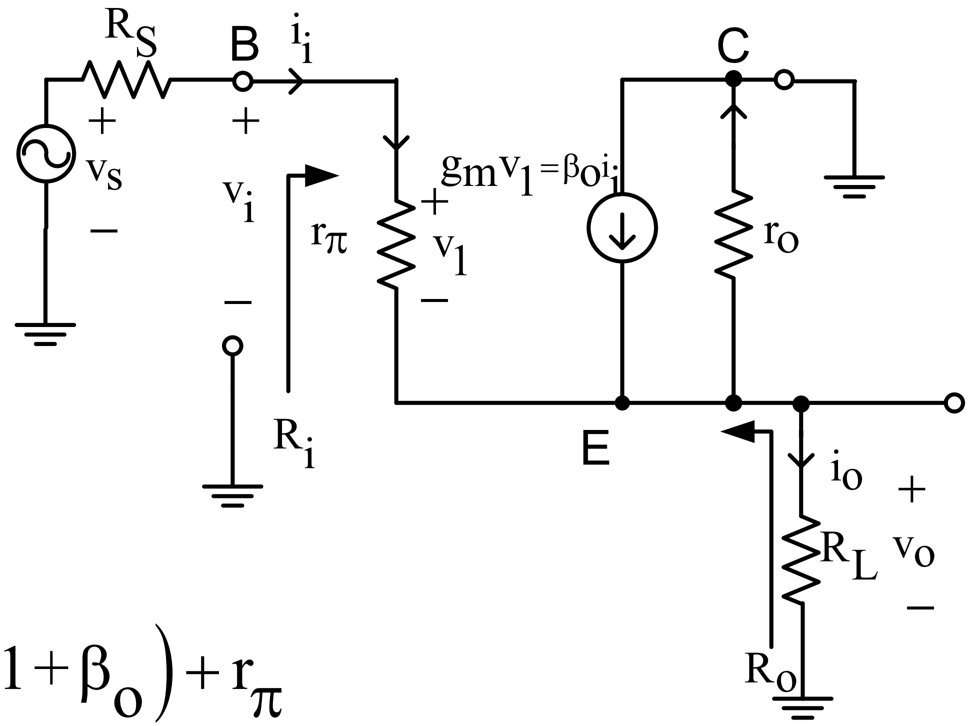
$$i_i(1+\beta_o) = (v_i - i_i r_\pi)\left(\frac{1}{R_L} + \frac{1}{r_o}\right)$$

$$i_i\left[1+\beta_o + r_\pi\left(\frac{1}{R_L} + \frac{1}{r_o}\right)\right] = v_i\left(\frac{1}{R_L} + \frac{1}{r_o}\right)$$

$$R_i = \frac{v_i}{i_i} = \frac{\left[1+\beta_o + r_\pi\left(\frac{1}{R_L} + \frac{1}{r_o}\right)\right]}{\left(\frac{1}{R_L} + \frac{1}{r_o}\right)} \quad \leftarrow \text{enough.}$$

Hence, this circuit is not unilateral as the input resistance depends on the load resistor R_L.

$$R_i = \frac{1+\beta_o + r_\pi\left(\frac{1}{R_L}+\frac{1}{r_o}\right)}{\left(\frac{1}{R_L}+\frac{1}{r_o}\right)}$$



For R_i with no load, i.e. $R_L = \infty$:

$$R_i = \left.\frac{v_i}{i_i}\right|_{R_L=\infty}$$

$$R_i = \left[1+\beta_o + r_\pi\left(\frac{1}{r_o}\right)\right]r_o = r_o\left(1+\beta_o\right)+r_\pi$$

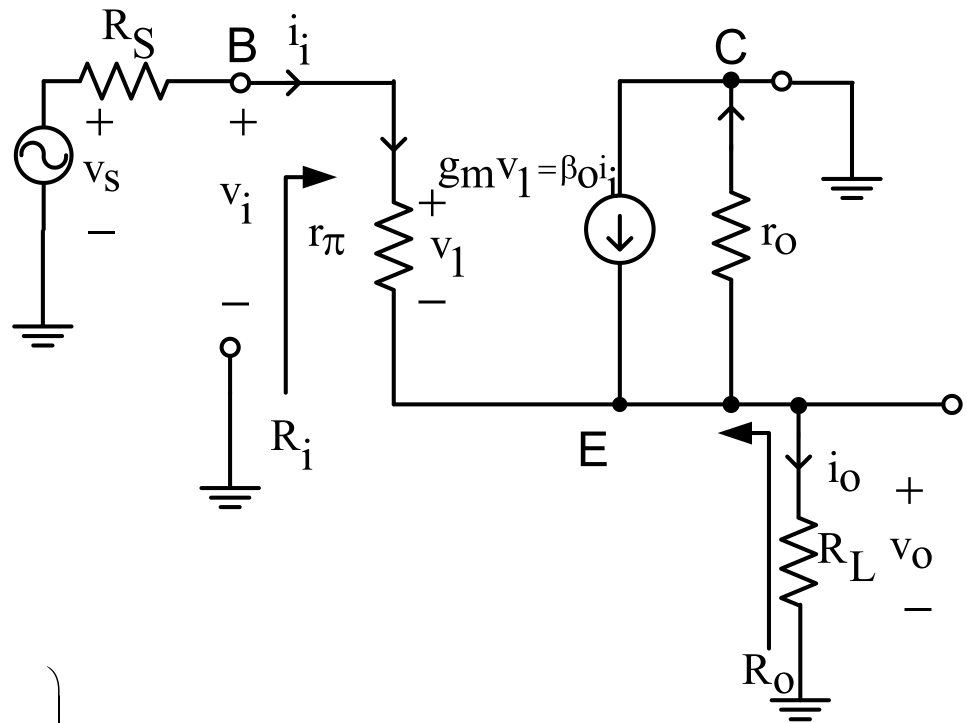
To determine a_v:

Overall voltage gain: $a_v = v_o / v_s$

At node E,

$$\frac{v_s - v_o}{R_s + r_\pi} + g_m v_1 = v_o\left(\frac{1}{R_L} + \frac{1}{r_o}\right)$$

$$\frac{v_s}{R_s + r_\pi} + g_m \frac{r_\pi (v_s - v_o)}{R_s + r_\pi} = v_o\left(\frac{1}{R_L} + \frac{1}{r_o} + \frac{1}{R_s + r_\pi}\right)$$



Since $g_m r_\pi = \beta_o$,

$$\frac{v_s}{R_s + r_\pi} + \frac{\beta_o (v_s - v_o)}{R_s + r_\pi} = v_o\left(\frac{1}{R_L} + \frac{1}{r_o} + \frac{1}{R_s + r_\pi}\right)$$

$$v_s\left(\frac{1}{R_s + r_\pi} + \frac{\beta_o}{R_s + r_\pi}\right) = v_o\left(\frac{1}{R_L} + \frac{1}{r_o} + \frac{1}{R_s + r_\pi} + \frac{\beta_o}{R_s + r_\pi}\right)$$

$$a_v = \frac{v_o}{v_s} = \frac{(1+\beta_o)\left(\frac{1}{R_s + r_\pi}\right)}{\left(\frac{1}{R_L} + \frac{1}{r_o} + \frac{1+\beta_o}{R_s + r_\pi}\right)} \qquad \leftarrow \text{enough.}$$

$$a_v = \frac{v_o}{v_s} = \frac{(1+\beta_o)\left(\frac{1}{R_S+r_\pi}\right)}{\left(\frac{1}{R_L}+\frac{1}{r_o}+\frac{1+\beta_o}{R_S+r_\pi}\right)}$$

$$= \frac{1}{\frac{1}{(1+\beta_o)\left(\frac{1}{R_S+r_\pi}\right)}\left(\frac{1}{R_L}+\frac{1}{r_o}+\frac{1+\beta_o}{R_S+r_\pi}\right)}$$

$$= \frac{1}{\left(\frac{R_S+r_\pi}{(1+\beta_o)R_L}+\frac{R_S+r_\pi}{(1+\beta_o)r_o}+1\right)}$$

$$= \frac{1}{\left(\frac{r_o(R_S+r_\pi)+R_L(R_S+r_\pi)}{(1+\beta_o)R_L r_o}+1\right)}$$

$$= \frac{1}{\left(\frac{(R_S+r_\pi)(r_o+R_L)}{(1+\beta_o)R_L r_o}+1\right)}$$

$$= \frac{1}{\left(\frac{(R_S+r_\pi)}{(1+\beta_o)R_L // r_o}+1\right)}$$

Open-circuit overall voltage gain, i.e. $R_L = \infty$:

$$a_v = \frac{(1+\beta_o)\left(\frac{1}{R_S+r_\pi}\right)}{\left(\frac{1}{r_o}+\frac{1+\beta_o}{R_S+r_\pi}\right)}$$

$$= \frac{1}{\frac{1}{(1+\beta_o)\left(\frac{1}{R_S+r_\pi}\right)}\left(\frac{1}{r_o}+\frac{1+\beta_o}{R_S+r_\pi}\right)}$$

Hence,

$$a_v = \left.\frac{v_o}{v_s}\right|_{R_L=\infty} = \frac{1}{\left(\frac{(R_S+r_\pi)}{(1+\beta_o)r_o}+1\right)}$$

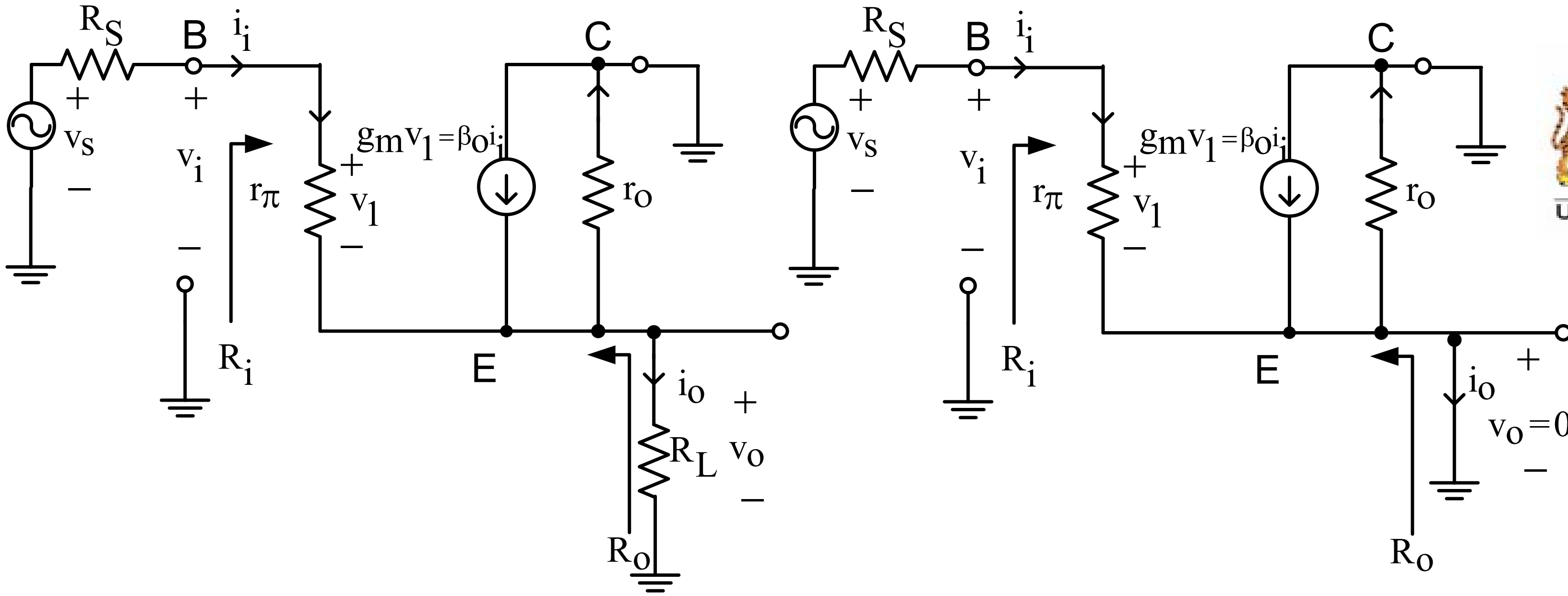
$$a_v = \frac{1}{\left(\frac{(R_s + r_\pi)}{(1+\beta_o)R_L // r_o} + 1\right)}$$

From the a_v expression, a_v will always be less than unity.

If $\beta_o(R_L // r_o) >> R_s + r_\pi$, then $a_v \approx 1$. This means that the output signal follows the input signal. Hence, this topology is also known as the emitter follower.

If $r_\pi >> R_S$, $\beta_o >> 1$ and $r_o >> R_L$, then $a_v = \frac{1}{\left(\frac{r_\pi}{(\beta_o)R_L} + 1\right)}$

Since $g_m r_\pi = \beta_o$, then $a_v = \frac{1}{\left(\frac{1}{g_m R_L} + 1\right)} = \frac{g_m R_L}{(1 + g_m R_L)}$



To determine the short circuit current gain, a_i :

$$a_i = \left.\frac{i_o}{i_i}\right|_{v_o=0}$$

$$i_o = i_i + g_m v_1$$

$$v_1 = i_i r_\pi$$

$$i_o = i_i + g_m i_i r_\pi$$

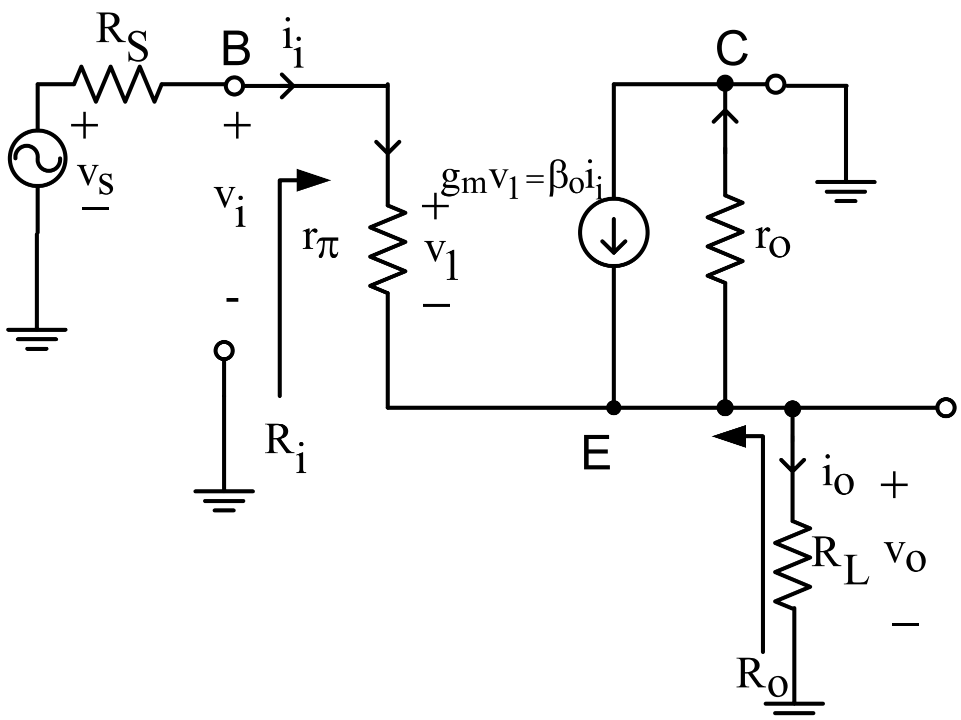
$$a_i = \frac{i_o}{i_i} = \frac{i_i\left(1+g_m r_\pi\right)}{i_i} = 1 + g_m r_\pi = 1 + \beta_o$$

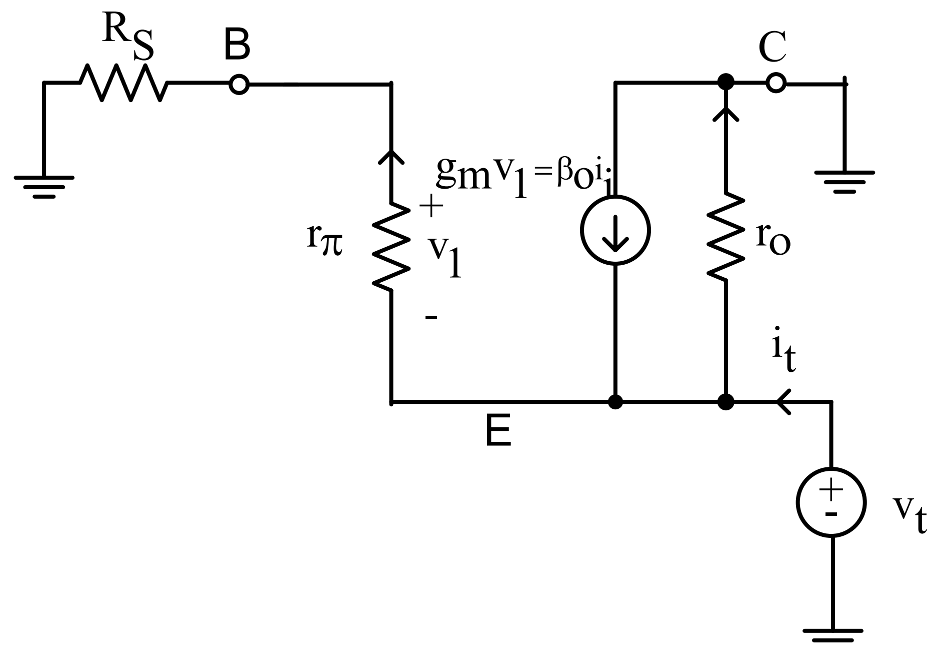
or

$$g_m v_1 = g_m i_i r_\pi = \beta_o i_i$$

$$i_o = i_i + \beta_o i_i = i_i\,(1 + \beta_o\,)$$

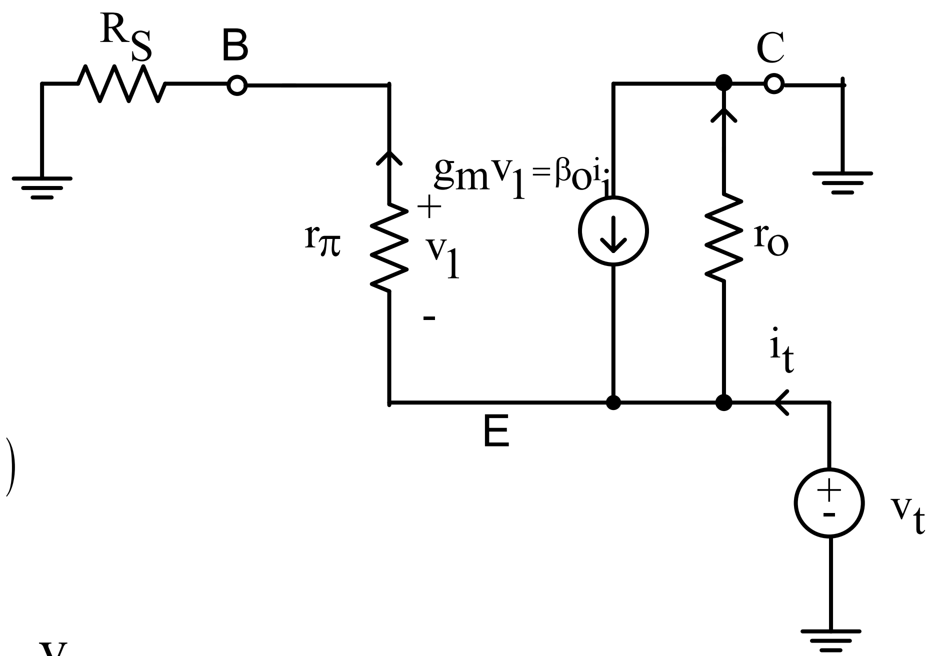
$$a_i = i_o\,/\,i_i = 1 + \beta_o$$





To determine R_o :

$$R_o = \left.\frac{v_t}{i_t}\right|_{v_s=0}$$



$$v_1 = -\frac{r_\pi}{R_S + r_\pi}(v_t)$$

At node E,

$$i_t + g_m v_1 = \frac{v_t}{r_o} + \frac{v_t}{R_S + r_\pi}$$

$$i_t = \frac{v_t}{r_o} + \frac{v_t}{R_S + r_\pi} + \frac{g_m r_\pi v_t}{R_S + r_\pi}$$

$$R_o = \frac{v_t}{i_t} = \frac{1}{\frac{1}{r_o} + \frac{1+\beta_o}{R_S + r_\pi}} \quad \leftarrow \text{enough}$$

$$= \left(\frac{R_S + r_\pi}{1+\beta_o}\right) \Bigg\| r_o$$

Hence, this circuit is not unilateral as the output resistance depends on the source resistance R_S.

$$R_o = \left(\frac{R_S + r_\pi}{1+\beta_o} \right) \Big\| r_o$$

$$= \left(\frac{R_S}{1+\beta_o} + \frac{r_\pi}{1+\beta_o} \right) \Big\| r_o$$

If β_o >> 1, then $R_o = \left(\frac{R_S}{1+\beta_o} + \frac{r_\pi}{\beta_o} \right) \Big\| r_o$

Since $g_m r_\pi = \beta_o$, then $R_o = \left(\frac{R_S}{1+\beta_o} + \frac{1}{g_m} \right) \Big\| r_o$

If $r_o >> \frac{R_S}{1+\beta_o} + \frac{1}{g_m}$, then $R_o = \frac{R_S}{1+\beta_o} + \frac{1}{g_m}$

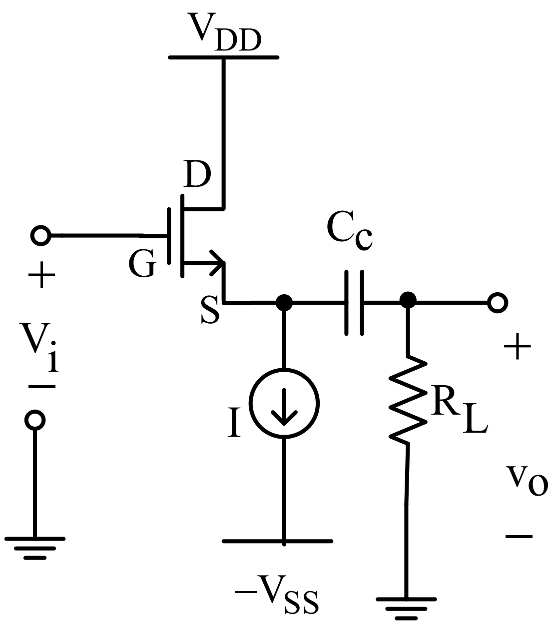
$$R_o = \frac{R_S}{1+\beta_o} + \frac{1}{g_m}$$

$$R_i = r_\pi + (1+\beta_o)(R_L \| r_o)$$

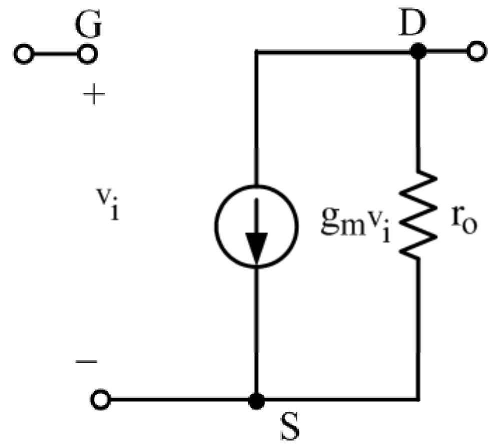
$$a_v = \frac{1}{1 + \frac{R_S + r_\pi}{(1+\beta_o)(R_L \| r_o)}} \approx 1$$

The emitter follower has high i/p resistance, low o/p resistance and near-unity voltage gain. Therefore, it is widely used as an impedance transformer to reduce loading of a preceding signal source by the i/p impedance of a following stage.

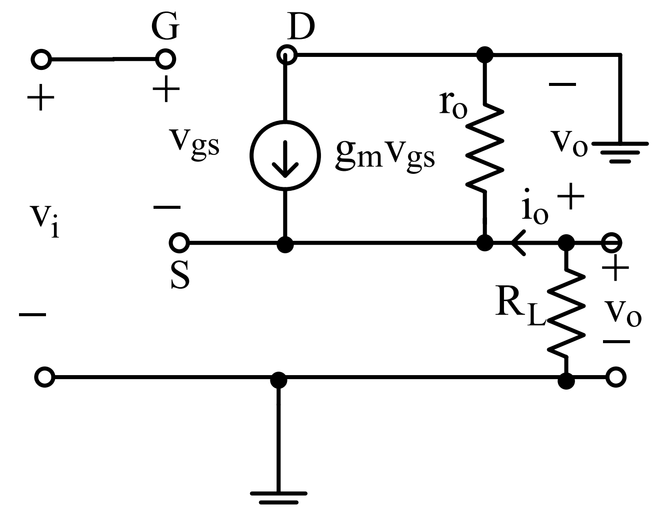
Common-drain configuration (source follower)



Small-signal for FET



Small-signal for CD



I/p signal applied to G. O/p signal taken from S.

To determine the open-circuit voltage gain, a_v :

$$a_v = \left.\frac{v_o}{v_i}\right|_{R_L=\infty, i_o=0}$$

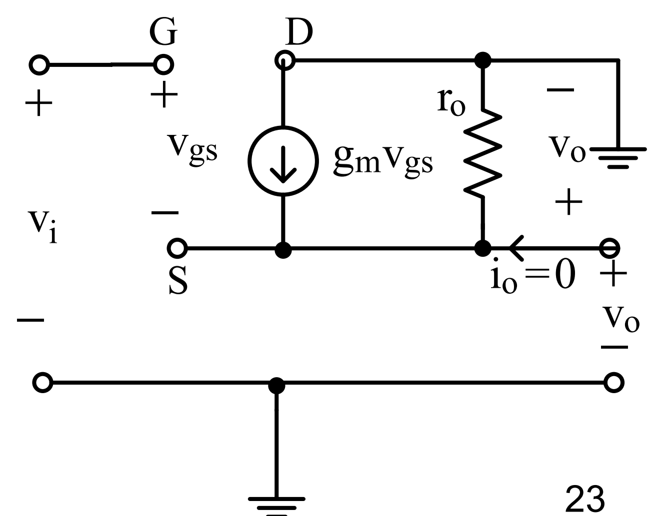
From KVL around the i/p loop,

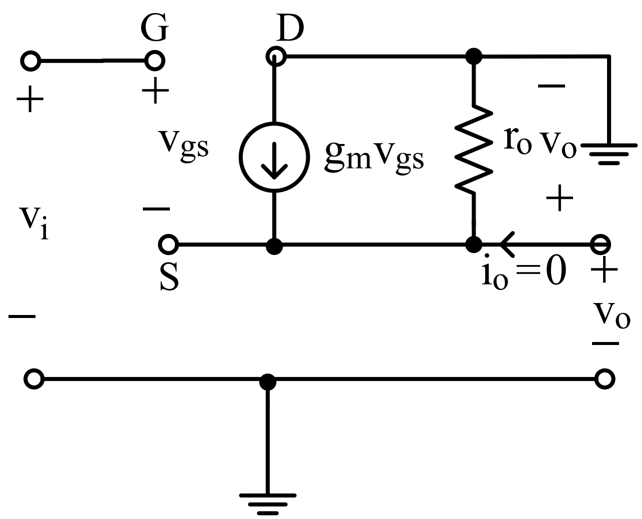
$-v_i + v_{gs} + v_o = 0$

$v_i = v_{gs} + v_o$

KCL at the o/p node:

$g_m v_{gs} = v_o / r_o$ for $R_L = \infty$





$$g_m v_{gs} = v_o / r_o$$

$$v_i = v_{gs} + v_o$$

$$g_m v_i = v_o\left(\frac{1}{r_o} + g_m\right)$$

$$a_v = \frac{v_o}{v_i} = \frac{g_m}{\left(\frac{1}{r_o} + g_m\right)} \quad \leftarrow \text{enough}$$

$$= \frac{g_m r_o}{1 + g_m r_o}$$

If r_o is finite, this gain is < 1.

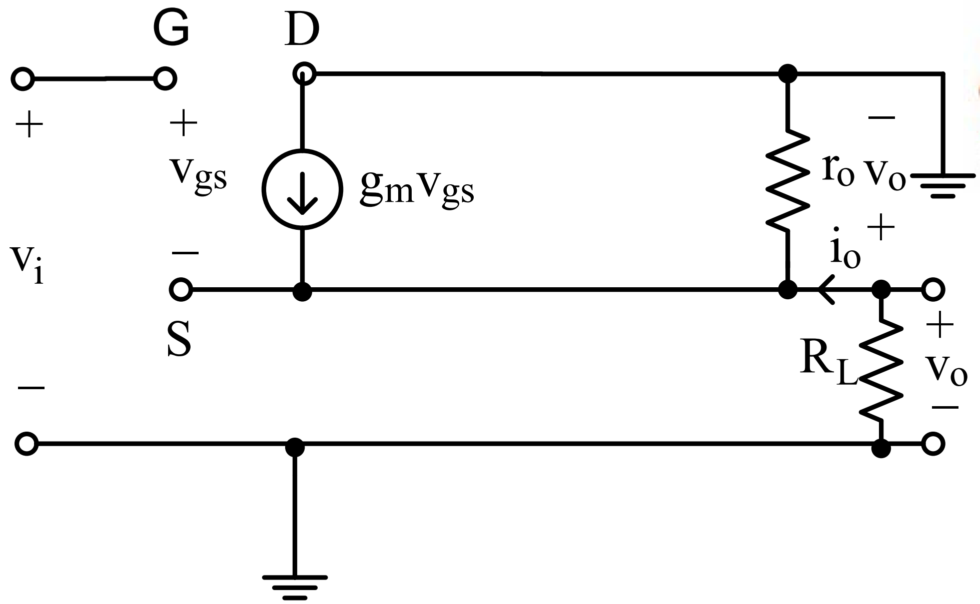
$$a_v = \frac{g_m}{\left(\frac{1}{r_o} + g_m\right)}$$

If $r_o \to \infty$, then

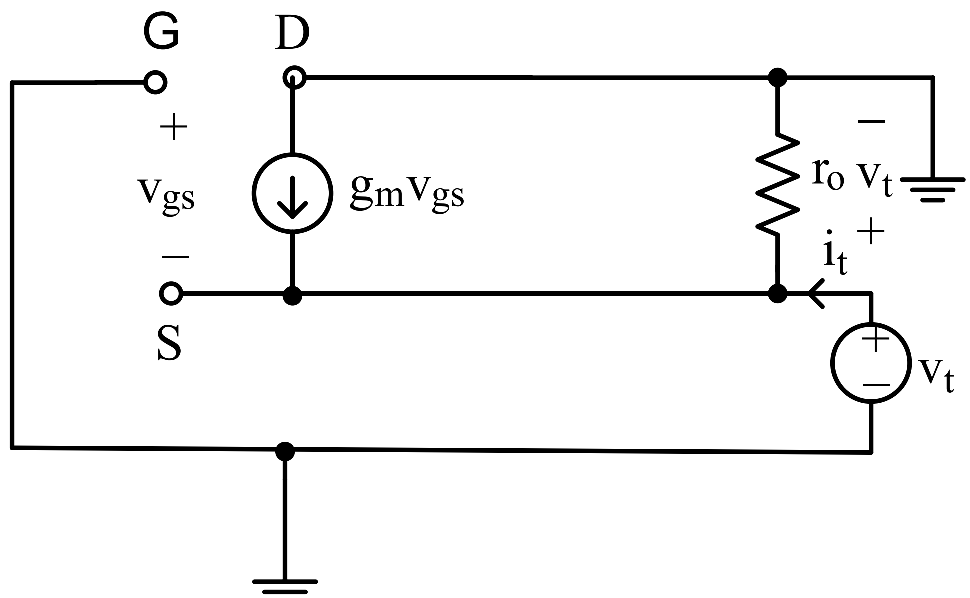
$$a_v = \frac{g_m}{g_m} = 1$$

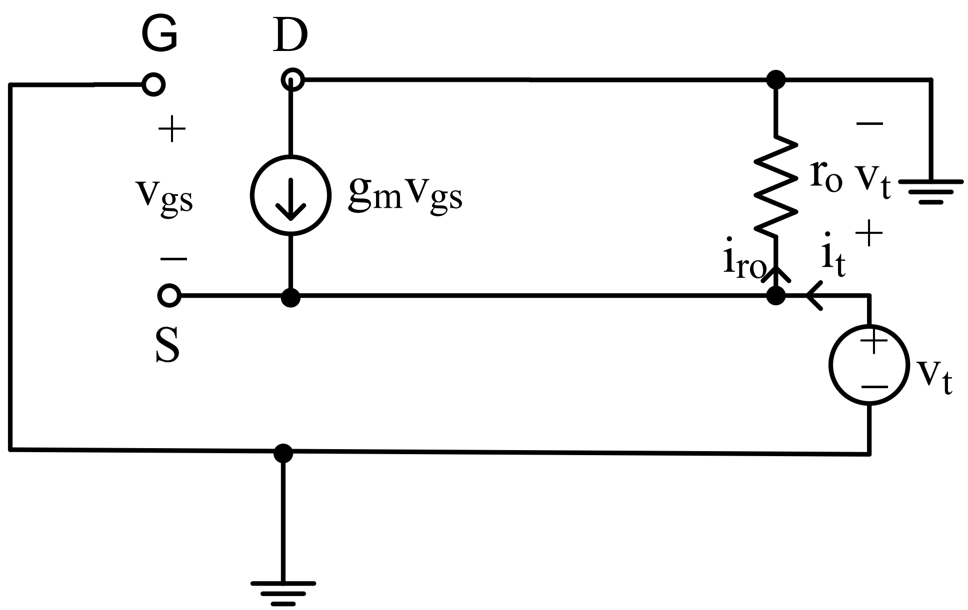
To determine R_o :

$$R_o = \left.\frac{v_t}{i_t}\right|_{v_i=0}$$



The output resistance can be calculated by setting v_i = 0 and driving the output with a voltage source v_t.





$v_{gs} = -v_t$
$v_{gs} = v_g - v_s = -v_s$

At the S node,

$$g_m v_{gs} + i_t = \frac{v_t}{r_o}$$

$$i_t = \left(\frac{1}{r_o} + g_m\right) v_t$$

$$R_o = \frac{v_t}{i_t} = \frac{1}{\left(\frac{1}{r_o} + g_m\right)}$$

If $r_o \to \infty$,

$$R_o = \frac{1}{g_m}$$

	CE	CB	CC
R_i	$r_\pi = \frac{\beta_o}{g_m}$	$r_e = \frac{1}{g_m + \frac{1}{r_\pi}} = \frac{1}{g_m\left(1+\frac{1}{\beta_o}\right)} = \frac{\alpha_o}{g_m}$	$r_\pi + (\beta_o + 1)(R_L \Vert r_o)$
G_m	g_m	g_m	$\frac{1+\beta_o}{R_S + r_\pi}$
R_o	$R_C \Vert r_o$	R_C	$\frac{R_S + r_\pi}{1+\beta_o} \Vert r_o$
a_v	$-g_m(R_C \Vert r_o)$	$g_m R_C$	$\frac{1}{1+\frac{R_S + r_\pi}{(1+\beta_o) r_o \Vert R_L}}$
a_i	β_o	$g_m r_e = \alpha_o$	$1+ \beta_o$

$$r_\pi = v_i / i_b = v_i \beta_o / i_c .$$

Since $g_m = i_c/v_i$,

$$r_\pi = \beta_o / g_m = \beta_o V_T / I_C$$

	CE	CB	CC
R_i	medium	↓	↑
R_o	medium	↑	↓
a_v	↑	↑	≤ 1
a_i	↑	≤ 1	↑
$a_p = a_v\, a_i$	↑	$\approx a_v$	$\approx a_i$
i/p–o/p phase shift (voltage)	180°	0°	0°

	CS	CG	CD
R_i	∞	$\frac{1}{g_m}$	∞
G_m	g_m	g_m	g_m
R_o	$R_D \Vert r_o$	R_D	$\frac{1}{g_m + \frac{1}{r_o}}$
a_v	$-g_m(R_D \Vert r_o)$	$g_m R_D$	$\frac{g_m r_o}{1+g_m r_o + \frac{r_o}{R_L}}$
a_i	∞	1	∞

Open-circuit voltage gain ($R_L = \infty$),

$$a_{vo} = \frac{v_o}{v_i} = \frac{v_o}{i_o} \times \frac{i_o}{v_i} = R_o G_m$$

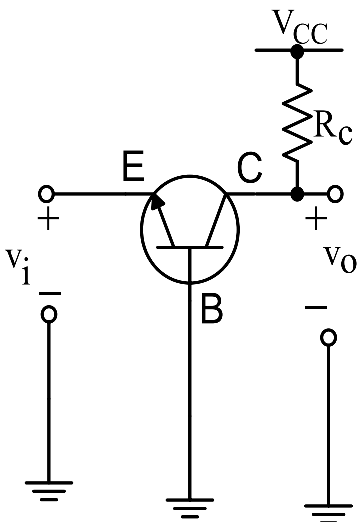
	CS	CG	CD
R_i	∞	↓	∞
R_o	medium	↑	↓
a_v	↑	↑	< 1
a_i	∞	1	∞
$a_p = a_v\, a_i$	∞	$\approx a_v$	∞
i/p–o/p phase shift	180° (voltage)	0°	0°

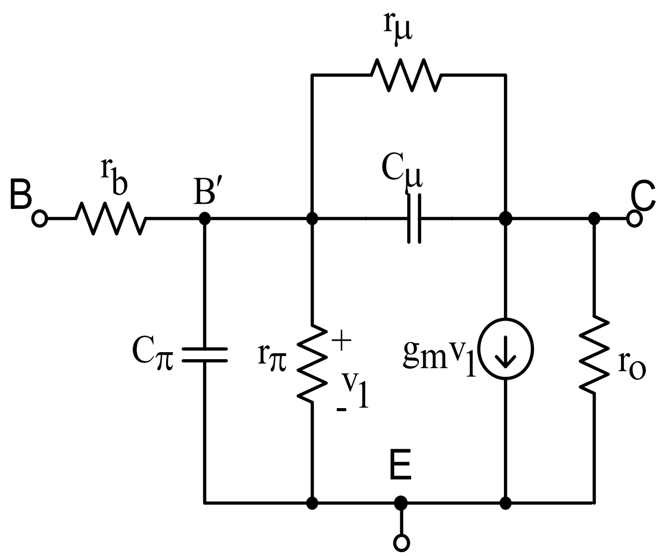
Short-circuit current gain ($R_L = 0$),

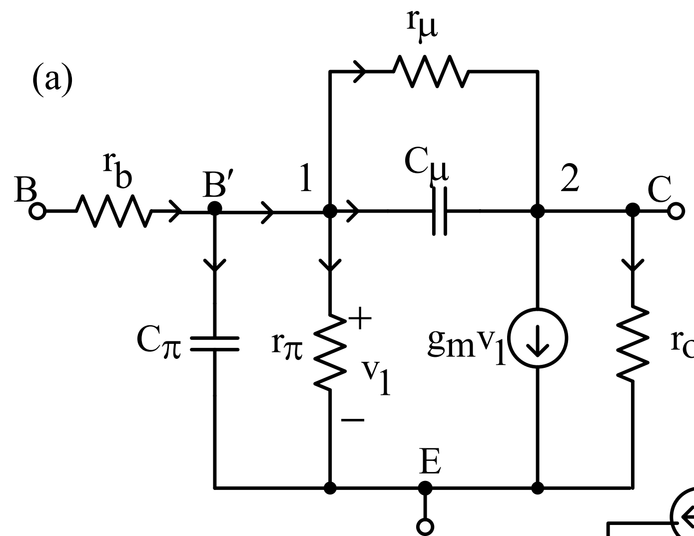
$$a_{is} = \frac{i_o}{i_i} = \frac{i_o}{v_i} \times \frac{v_i}{i_i} = R_i G_m$$

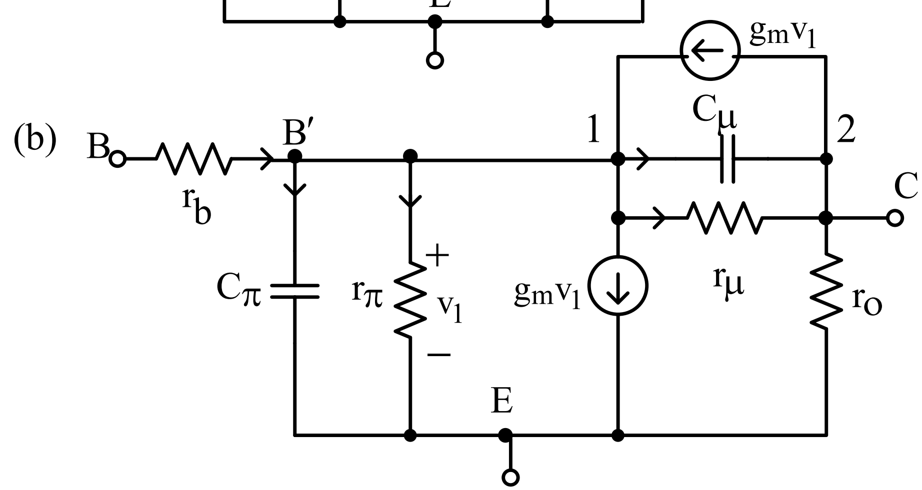
To generate a T-model for a BJT from a hybrid-π model .

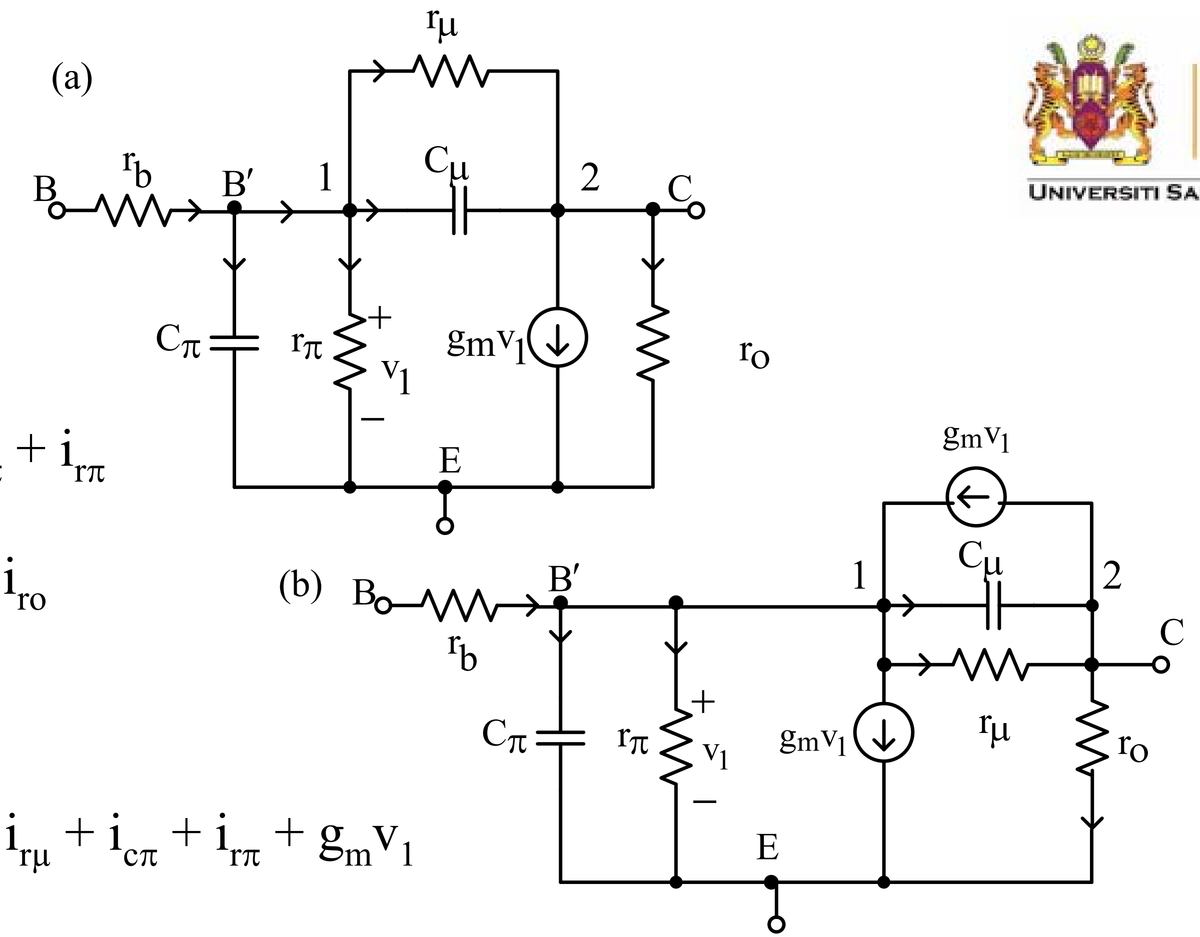
The analysis of CB amplifiers can be simplified if the model is changed from a hybrid-π to a T-model.











<u>Figure (a):</u>

Node 1: $i_{rb} = i_{c\mu} + i_{r\mu} + i_{c\pi} + i_{r\pi}$

Node 2: $i_{c\mu} + i_{r\mu} = g_m v_1 + i_{ro}$

<u>Figure (b):</u>

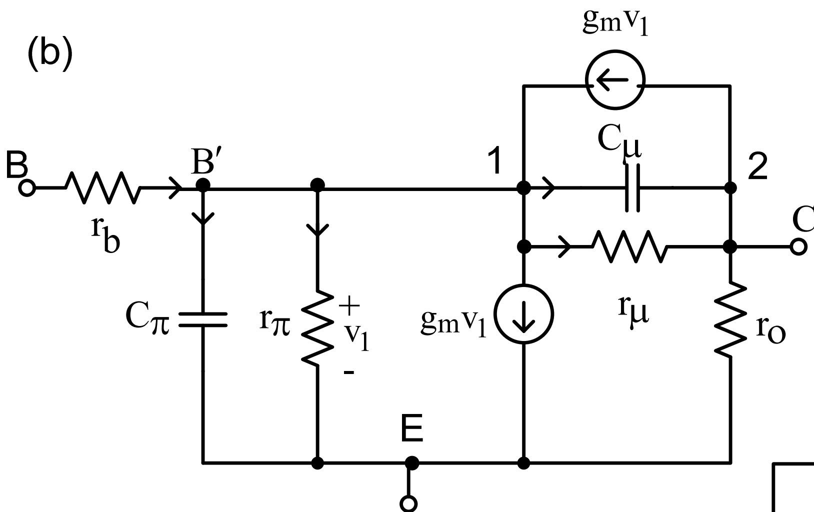
Node 1: $i_{rb} + g_m v_1 = i_{c\mu} + i_{r\mu} + i_{c\pi} + i_{r\pi} + g_m v_1$

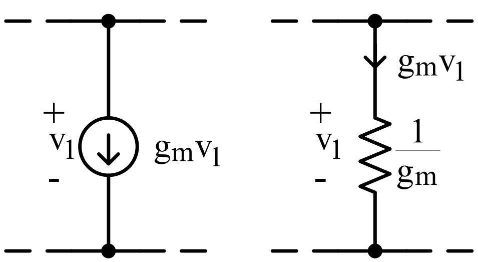
$i_{rb} = i_{c\mu} + i_{r\mu} + i_{c\pi} + i_{r\pi}$

Node 2: $i_{c\mu} + i_{r\mu} = g_m v_1 + i_{ro}$

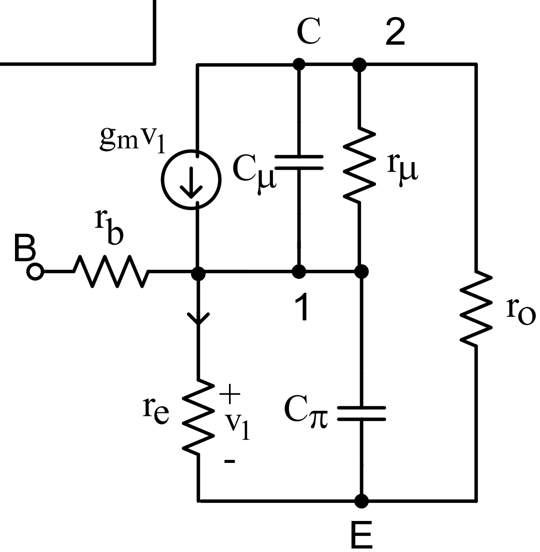
Hence, the change from Figure (a) to Figure (b) does not affect the current flowing in the B.

(b)





(c)



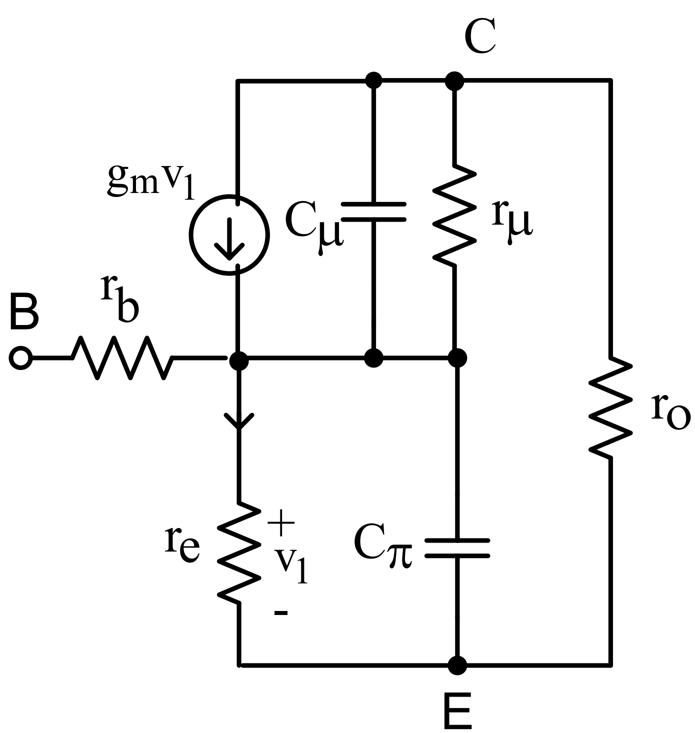
$$r_e = r_\pi // \frac{1}{g_m}$$

$$= \frac{r_\pi \left(\frac{1}{g_m}\right)}{r_\pi + \frac{1}{g_m}}$$

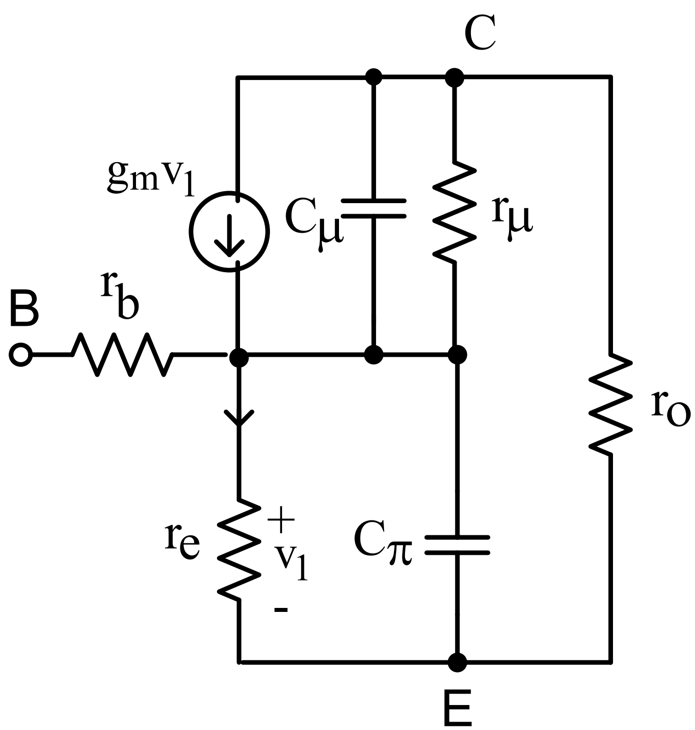
$$= \frac{r_\pi}{1 + g_m r_\pi}$$

$$r_\pi = \frac{\beta_o}{g_m}$$

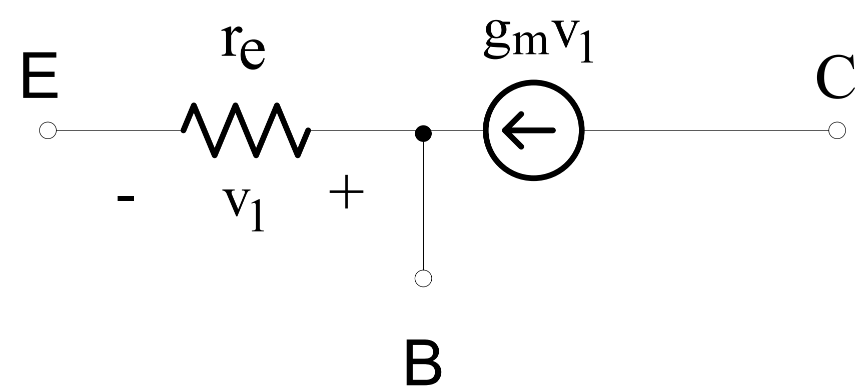
$$r_e = \frac{r_\pi}{1+g_m r_\pi} = \frac{\beta_o}{g_m\left(1+g_m\frac{\beta_o}{g_m}\right)} = \frac{1}{g_m}\frac{\beta_o}{1+\beta_o} = \frac{\alpha_o}{g_m}$$



At low freqs., C_π and C_μ are neglected. Assume $r_b \approx 0$ and $r_o \approx \infty$. Hence, $r_\mu \approx \infty$ as $r_\mu = \beta_o r_o$.



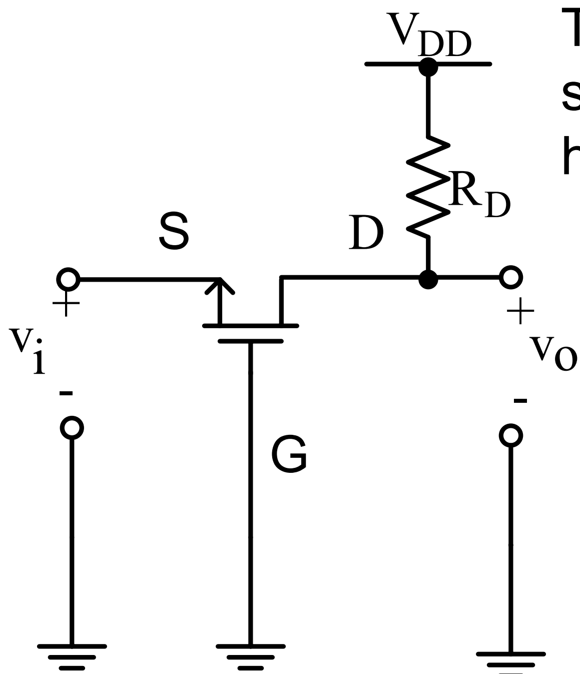
The T-model at low freq.:



When $r_o \approx \infty$, the circuit is said to be unilateral as there is no feedback from o/p (C) to i/p (E).

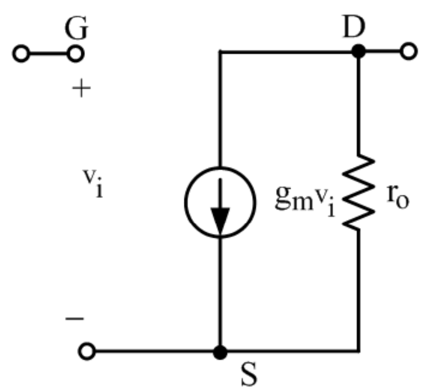
Generating a T-model for an FET from a hybrid-π model.

The analysis of CG amplifiers can be simplified if the model is changed from a hybrid-π to a T-model.

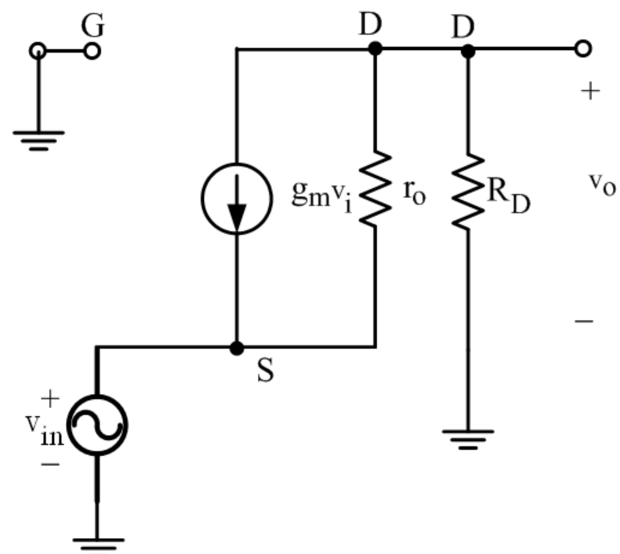


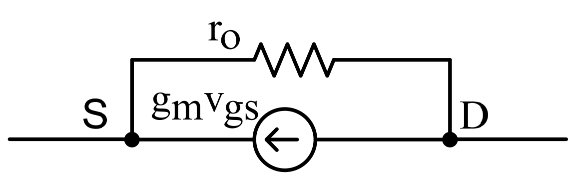
CG amplifier

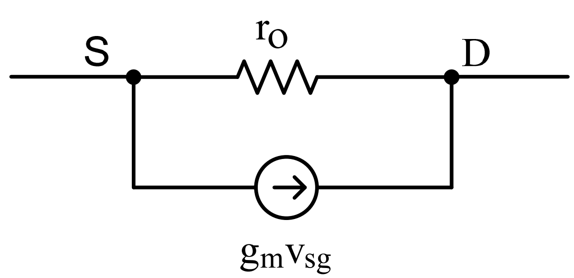
hybrid-π of an FET

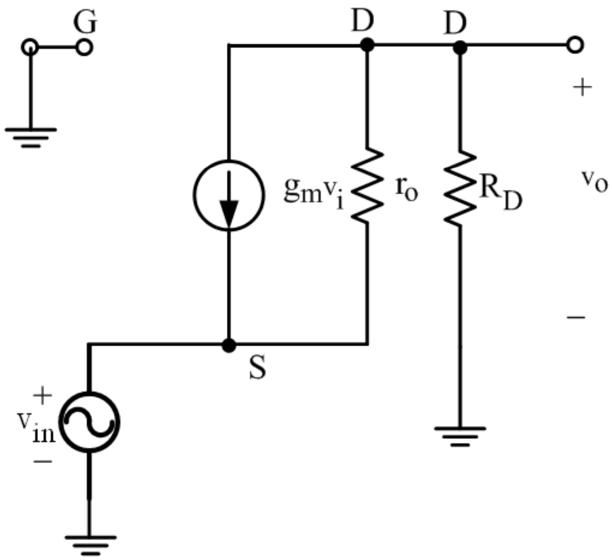


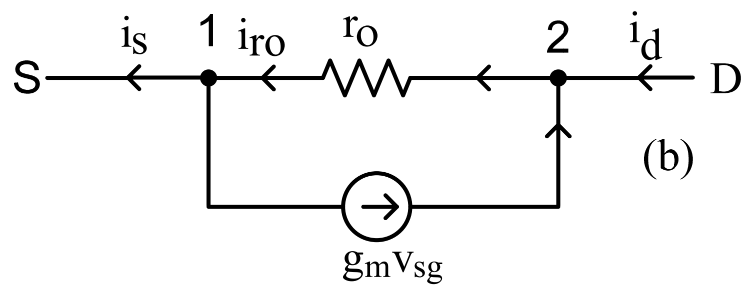
hybrid-π of a CG

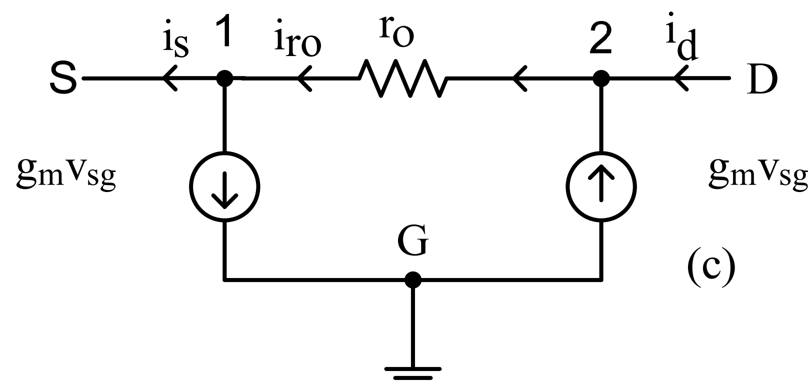












<u>Figure (b):</u>

Node 1: $i_{ro} = i_S + g_m v_{sg}$

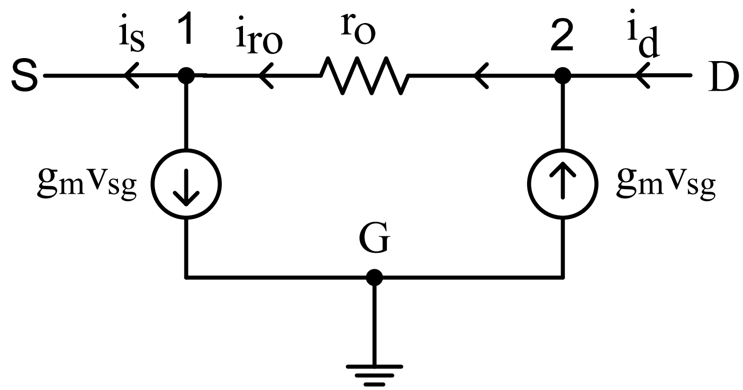
Node 2: $i_d + g_m v_{sg} = i_{ro}$

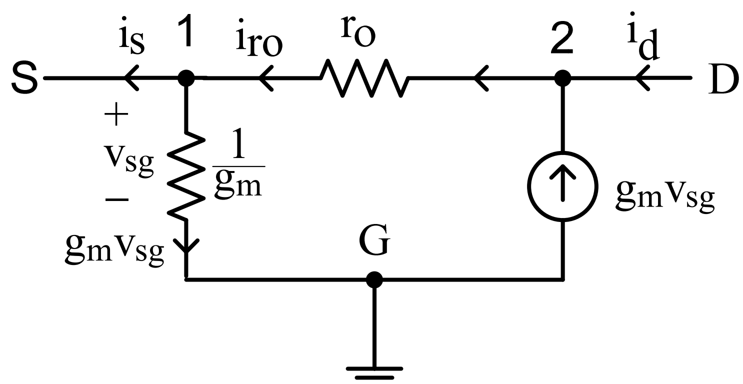
<u>Figure (c):</u>

Node 1: $i_{ro} = i_S + g_m v_{sg}$

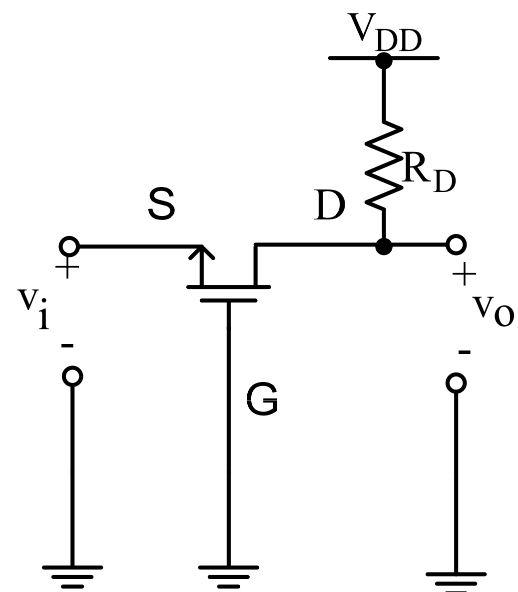
Node 2: $i_d + g_m v_{sg} = i_{ro}$

Equal currents are pushed into and pulled out of the G as the equations that describe the operation of the circuits are identical.





If r_o is finite, the circuit is bilateral because of the feedback. If $r_o \to \infty$, the cct. is unilateral.



For $r_o \to \infty$: